

# Waterfield Primary School

## Progression in Calculation Policy



January 2009

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## Rationale

At Waterfield, we believe a clear progression in calculation will support the learning and teaching of maths throughout the school, allow clarity and provide a secure foundation upon which to build and develop mathematical skills.

This policy contains the key pencil and paper procedures that will be taught within Waterfield. Although the focus of the policy is on pencil and paper procedures it is important to recognise that the ability to calculate mentally lies at the heart of the numeracy framework. In every written method there is an element of mental processing. Sharing written methods with the teacher encourages children to think about the mental strategies that underpin them and develop new ideas. Therefore, written recording both helps children to extend and clarify their thinking.

Children should be encouraged to see mathematics as both a written and a spoken language. Teachers should support and guide children through the following stages:

- Develop the use of pictures and use of words and symbols to represent numerical activities.
- Use standard symbols and conventions.
- Use of jottings to aid a mental strategy.
- Use of pencil and paper procedures.
- Use of a calculator.

It is important that children do not abandon jottings and mental methods once other pencil and paper procedures are introduced. Children will always be encouraged to look at a calculation/problem and then decide on the best method. **Progression in calculation is a developmental skill that should be taught when the child is ready.** Children's advancement in calculation should be at an appropriate time for their ability, which may not meet national expectations for their age.

Progression in calculation should include:

- A range of mental strategies to be used as a first resort, even once written methods have been introduced and embedded.
- An ability to understand and use the relationships between the four operations of number.
- An ability to explain, describe and record their methods.
- An ability to estimate and check whether the answer is correct.
- An ability to solve a wide range of problems involving calculation in a wide variety of contexts.
- An ability to choose and use the most appropriate method of calculation; mental, jottings, written or using a calculator.
- An ability to take the initiative to return to an earlier method that children are more confident with.

All methods should be taught with understanding rather than by rote and put into real life contexts.

This policy is divided into seven parts:

- Useful ITP's
- Progression from mental to written methods for addition
- Progression from mental to written methods for subtraction
- Progression from mental to written methods for multiplication
- Progression from mental to written methods for division
- Methods for solving time problems
- Methods for solving word problems

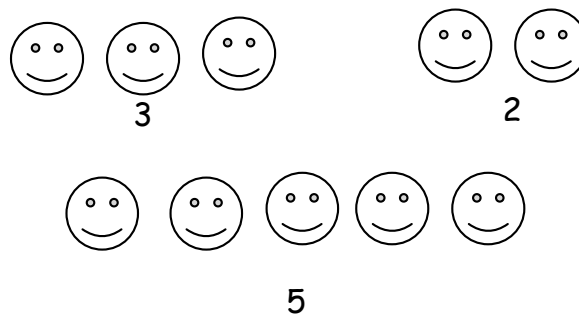
Teachers should liaise with previous class teachers to find out which method children have been using in each operation. This will ensure that the appropriate progression of methods will be followed into subsequent year groups.

## Progression from Mental to Written Methods for Addition

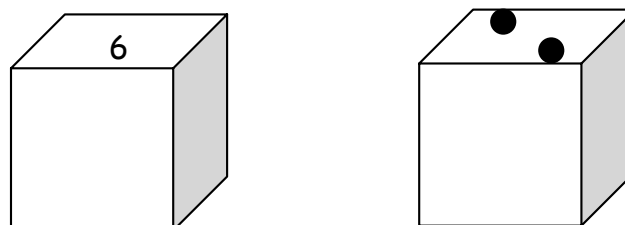
### Pre-requisite Skill:

- To be able to count reliably including one to one correspondence

1. Be able to count on from any number, combining two groups.

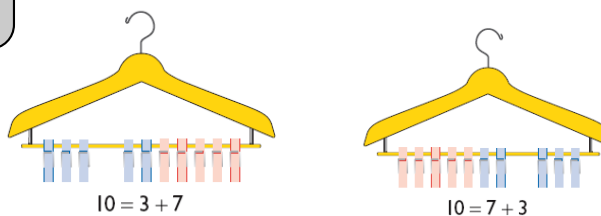


→  
To 100 and  
beyond



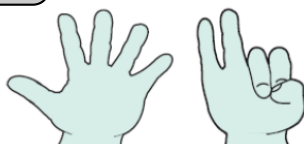
6 and 2 more makes 8  
 $6 + 2 = 8$

2. Recognise that addition can be done in any order.



Flip-flaps    Bead Strings

3. Know addition facts to ten.



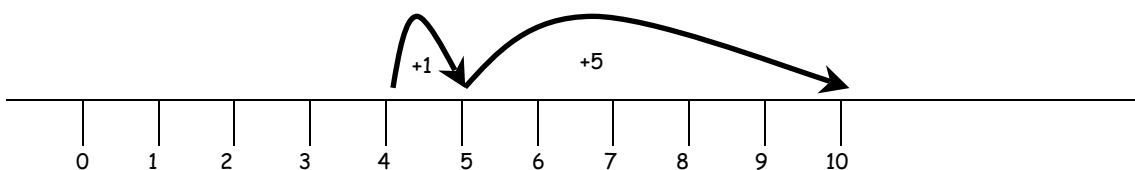
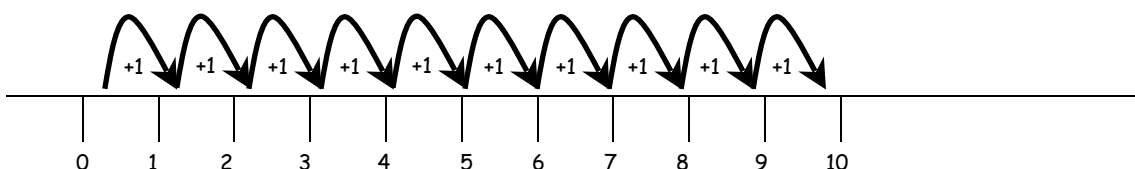
7 fingers up  
How many are down?

## Progression from Mental to Written Methods for Addition

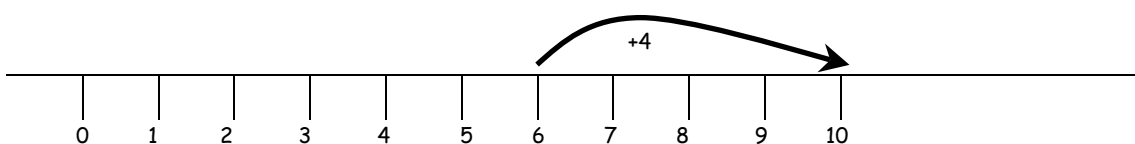
4.

Be able to count on in ones on a numbered line.  
Be able to bridge to the next ten (linked to the addition number facts).

$$4 + 6 = 10$$



More efficient jumps

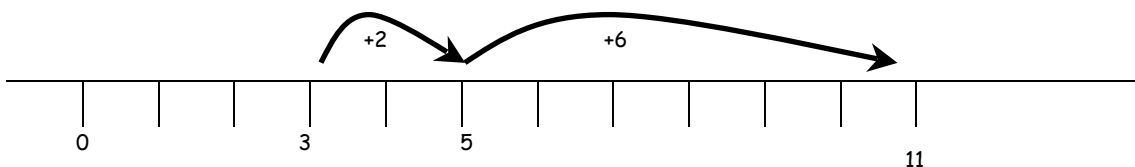


Larger number first

5.

Be able to count on using marked unlabelled number line, then an empty number line in jumps of various sizes, choosing jumps of a sensible size.

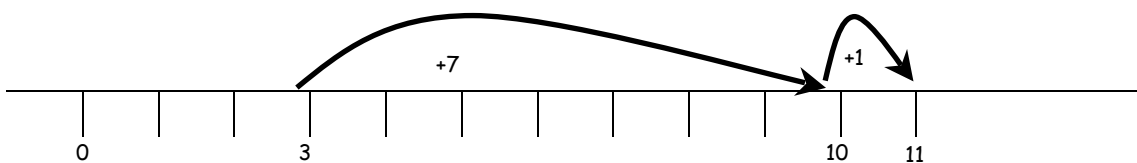
$$3 + 8 = 11$$



Bridging to 5

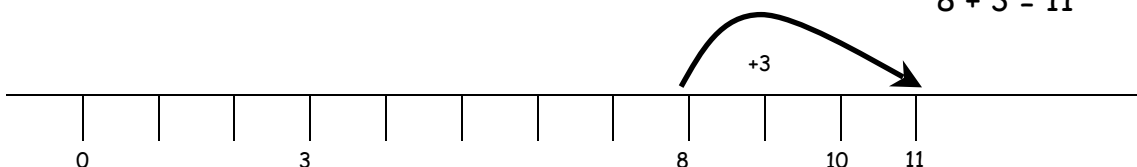
Progression from Mental to Written Methods for Addition

$$3 + 8 = 11$$

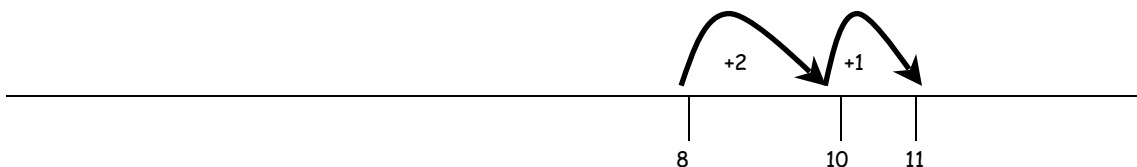


Bridging to 10

$$8 + 3 = 11$$



Larger number first

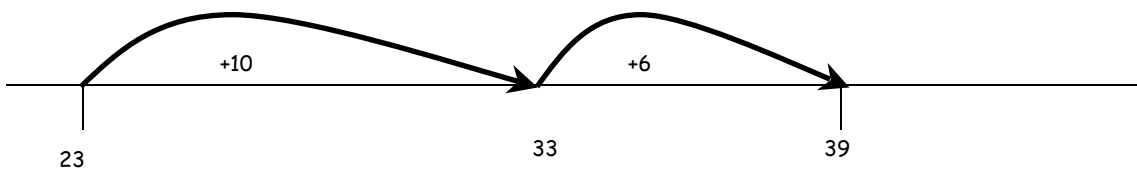


Empty number line

6.

Be able to partition two digit numbers. Be able to count on in tens and multiples of ten.

$$23 + 16 = 39$$



$$23 + 16 = 20 + 3 \text{ add } 10 + 6$$



$$30 + 9 = 39$$

## Progression from Mental to Written Methods for Addition

7.

Understand the importance of lining up the place value digits.

### Expanded Column Addition

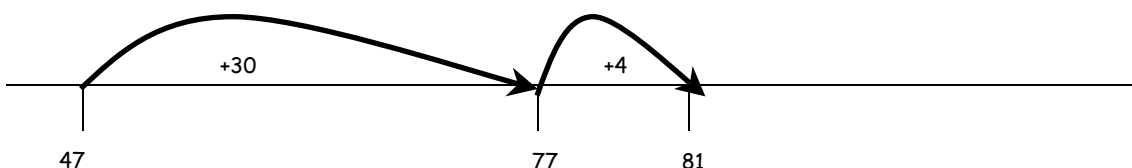
$$\begin{array}{r} \text{TU} \\ 23 \longrightarrow 20 + 3 \\ + 16 \longrightarrow \underline{10 + 6} \\ \hline 39 \longleftarrow 30 + 9 \end{array}$$

ITP Place Value

8.

Count on in multiples of ten and one, including carrying a ten.

$$47 + 34 = 47 + 30 + 4$$



$$\begin{aligned} 47 + 34 &= 40 + 7 \text{ add } 30 + 4 \\ &= 70 + 11 \\ &= 81 \end{aligned}$$

$$\begin{array}{r} \text{TU} \\ 47 \longrightarrow 40 + 7 \\ + 34 \longrightarrow \underline{30 + 4} \\ \hline 81 \longleftarrow \underline{70 + 11} \end{array}$$

$$\begin{array}{r} \text{TU} \\ 47 \longrightarrow 40 + 7 \\ + 34 \longrightarrow \underline{30 + 4} \\ \hline 81 \longleftarrow \underline{80 + 1} \\ 10 \end{array}$$

→  
Bridge the hundred:  
Bridge the ten and  
hundred

## Progression from Mental to Written Methods for Addition

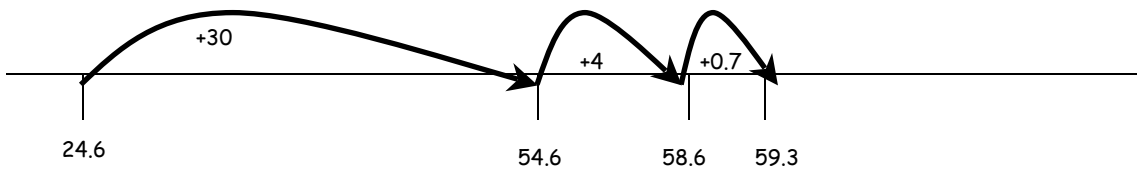
### Compact Method of Addition:

$\begin{array}{r} 40 + 7 \\ + 30 + 4 \\ \hline 70 + 11 \\ \hline \end{array}$	$\begin{array}{r} \text{TU} \\ 47 \\ + 34 \\ \hline 81 \\ 1 \end{array}$
---	--

9.

Extend to decimals

$$24.6 + 34.7 = 24.6 + 30 + 4 + 0.7$$



$$24.6 + 34.7 = 20 + 4 + 0.6 + 30 + 4 + 0.7$$

$50 + 8 + 1.3 = 59.3$

### Expanded Column Addition:

$\begin{array}{r} 24.6 \\ + 34.7 \\ \hline 59.3 \end{array}$	$\rightarrow$	$20 + 4 + 0.6$	$\rightarrow$	$24.6$	$\rightarrow$	$20 + 4 + 0.6$	$\rightarrow$	$+ 34.7$	$\rightarrow$	$+ 30 + 4 + 0.7$	$\rightarrow$	$59.3$	$\leftarrow$	$50 + 8 + 1.3$	$\leftarrow$	$59.3$	$\leftarrow$	$50 + 9 + 0.3$	$\leftarrow$	$1$
--	---------------	----------------	---------------	--------	---------------	----------------	---------------	----------	---------------	------------------	---------------	--------	--------------	----------------	--------------	--------	--------------	----------------	--------------	-----

### Compact Column Addition:

$$\begin{array}{r} 24.6 \\ + 34.7 \\ \hline 59.3 \\ 1 \end{array}$$

$\rightarrow$

Addition with two decimal places and beyond.

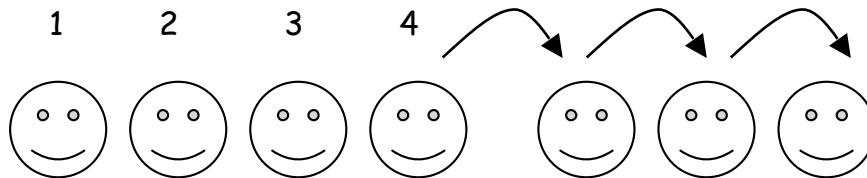
## Progression From Mental to Written Methods for Subtraction

*Prerequisite skill:*

- To be able to count reliably, including one to one correspondence.

1.

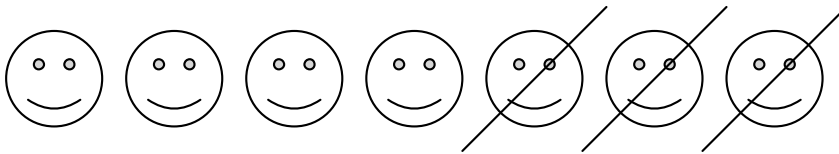
Be able to count up or back from any number



How many more do I need to make 7?

2.

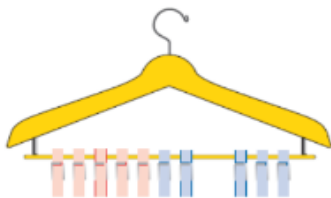
Understand subtraction as taking away



Remove some objects and count

3.

Know the inverse relationship of number facts to 10



$$7 + 3 = 10$$

$$10 - 3 = 7$$

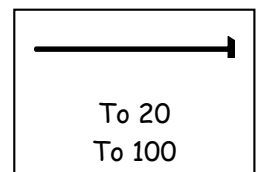


$$3 + 7 = 10$$

$$10 - 7 = 3$$



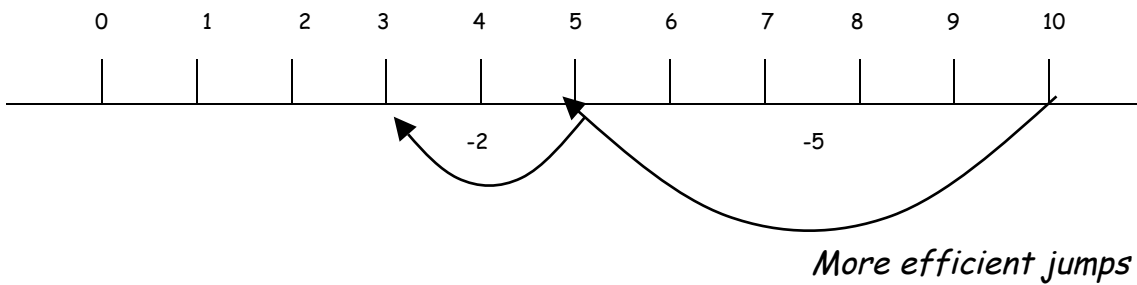
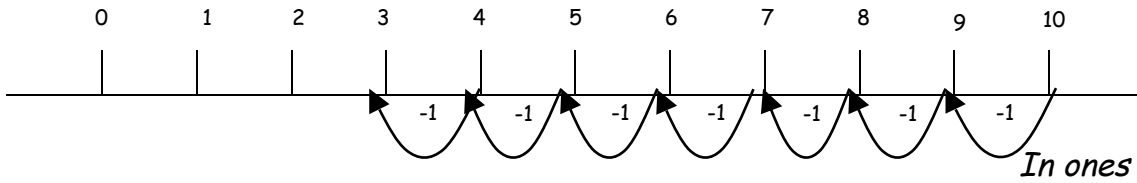
6 fingers up. How many are down?  
4 fingers down. How many are up?



## Progression From Mental to Written Methods for Subtraction

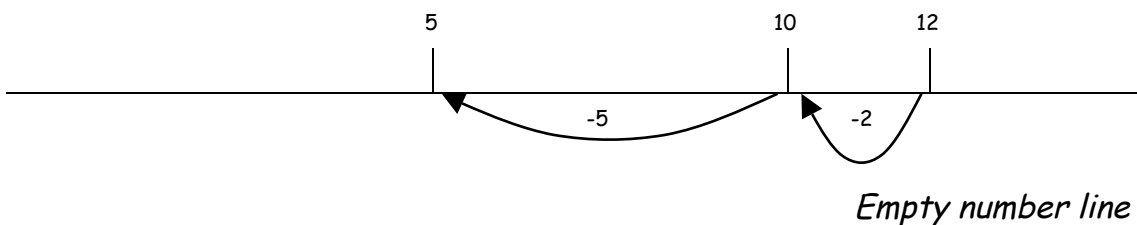
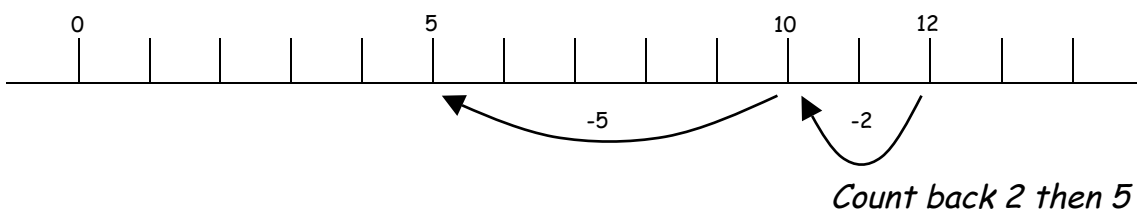
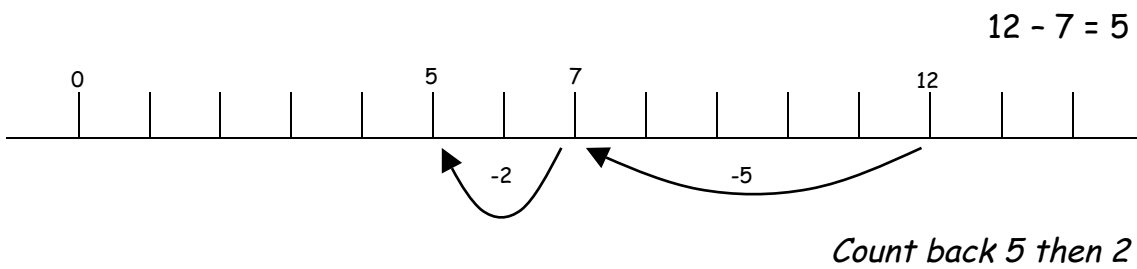
4.

Be able to count back in ones on a numbered line



5.

Be able to count back using a marked unlabelled number line, then an empty number line, in jumps of various sizes, choosing jumps of a sensible size.



## Progression From Mental to Written Methods for Subtraction

6.

Understand the concepts of subtraction as:

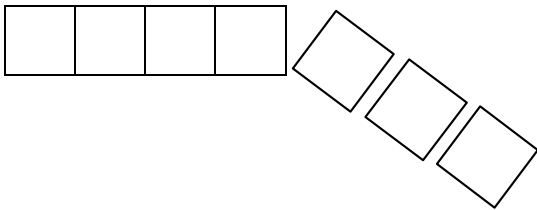
*Taking away*

*Finding the difference*

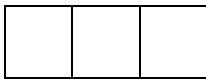
*Counting up*

*Counting back*

$$7 - 3 = 4$$



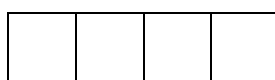
One quantity with 3 removed



Two quantities: 4 more make them the same  
(I have 3, how many more do I need to make 7?)



Three

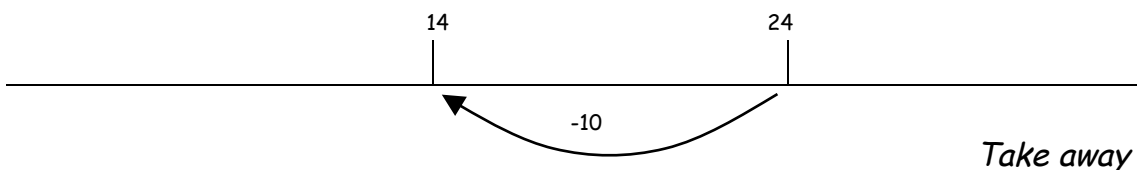


four more to make 7

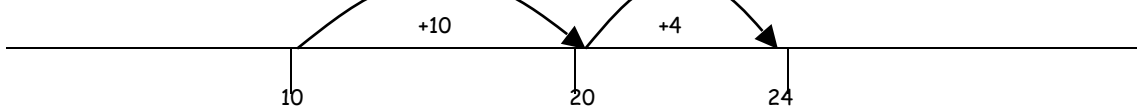
7.

Understand the link between subtraction as finding the difference and taking away

$$24 - 10 = 14$$



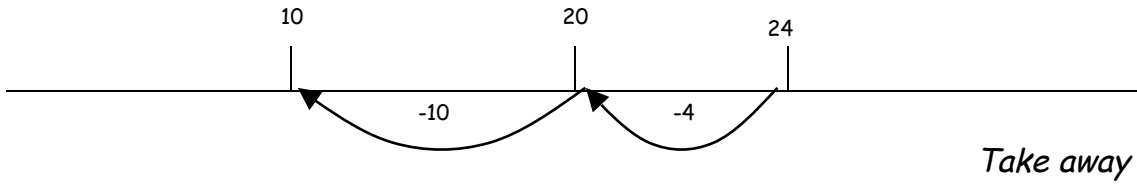
$$10 + ? = 24$$



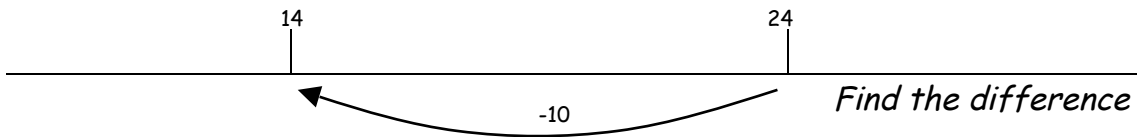
*Find the difference*

## Progression From Mental to Written Methods for Subtraction

$$24 - 14 = 10$$



$$24 + ? = 14$$



8.

Be able to subtract by decomposition

No carrying over

$$68 - 32 = 36$$

68	→	60	+	8
<u>- 32</u>	→	<u>- 30</u>	+	<u>- 2</u>
36	←	30	+	6

Carrying the tens to units

$$334 - 217 = 117$$

334	→	300	+	30	+	4
<u>- 217</u>	→	<u>- 200</u>	+	<u>- 10</u>	+	<u>- 7</u>
117						

↑	→	300	+	20	+	14
↑	→	<u>- 200</u>	+	<u>- 10</u>	+	<u>- 7</u>
↑	←	100		10		7

## Progression From Mental to Written Methods for Subtraction

### Carrying the hundreds to the tens

$$537 - 274 = 263$$

537	→	500	+	30	+	7
<u>- 274</u>	→	<u>- 200</u>	+	<u>- 70</u>	+	<u>- 4</u>
263						

↑	→	400	+	130	+	7
↑	→	<u>- 200</u>	+	<u>- 70</u>	+	<u>- 4</u>
		200		60		3

### Carrying tens to units and hundreds to tens

$$521 - 376 = 145$$

521	→	500	+	20	+	1
<u>- 376</u>	→	<u>- 300</u>	+	<u>- 70</u>	+	<u>- 6</u>
145						

↑	→	500	+	10	+	11
↑	→	<u>- 300</u>	+	<u>- 70</u>	+	<u>- 6</u>
↑	→	400	+	110	+	11
↑	→	<u>- 300</u>	+	<u>- 70</u>	+	<u>- 6</u>
		100		40		5

## Progression From Mental to Written Methods for Subtraction

9. Be able to subtract using the Compact Method

### Carrying tens to units

$$334 - 217 = 117$$

$$\begin{array}{r} 3^2 \cancel{3}^1 4 \\ - 217 \\ \hline 117 \end{array}$$

### Carrying hundreds to tens

$$537 - 274 = 263$$

$$\begin{array}{r} 4 \cancel{5}^1 3 7 \\ - 274 \\ \hline 263 \end{array}$$

### Carrying tens to units and hundreds to tens

$$534 - 378 = 156$$

$$\begin{array}{r} 4 \quad 1 \\ \cancel{5}^2 \cancel{3}^1 4 \\ - 378 \\ \hline 156 \end{array}$$

## Progression From Mental to Written Methods for Subtraction

### Carrying units to decimals

$$38.2 - 24.7 = 13.5$$

$$\begin{array}{r} 3\cancel{8}^{7} . 2 \\ - 24.7 \\ \hline 13.5 \end{array}$$

## Progression From Mental to Written Methods for Multiplication

*Prerequisite skill:*

- Addition skills to be reached before attempting multiplication.

1.

Be able to count in groups of 2, 5 and 10.



2



4



6



5



10



15



10



20

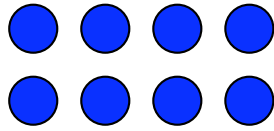


30

## Progression From Mental to Written Methods for Multiplication

2.

To understand double as two equal groups of objects or numbers

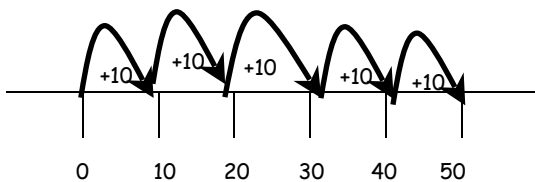
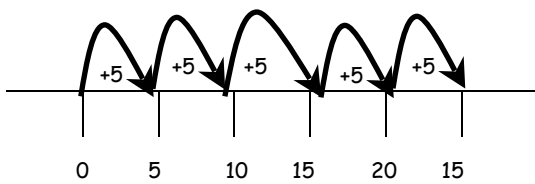
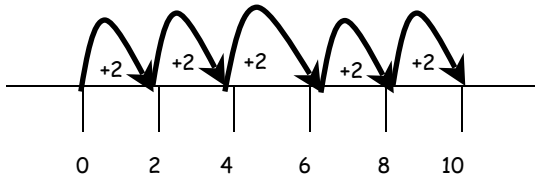


$$4 + 4 = 8$$

$$4 \times 2 = 8$$

3.


To represent jumps of 2, 5 and 10 on a number line




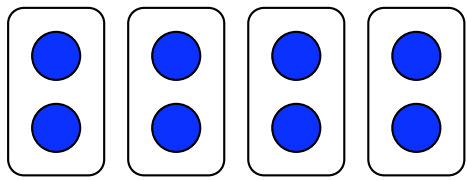
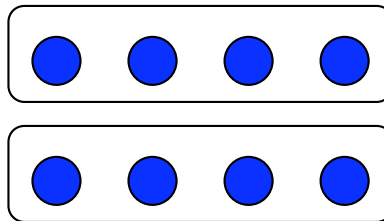
## Progression From Mental to Written Methods for Multiplication

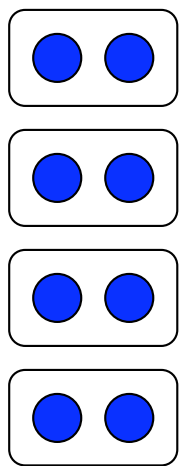
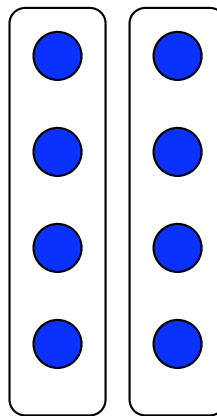
4.

To understand the commutative nature of multiplication

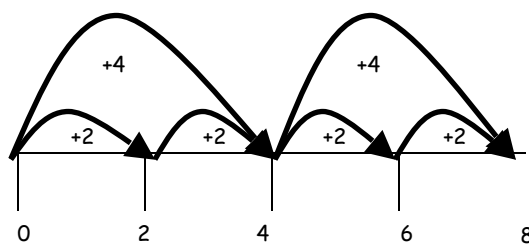

$$2 + 2 + 2 + 2 = 2 \times 4 = 8$$


$$4 + 4 = 4 \times 2 = 8$$


$$2 \times 4 = 8$$

$$4 \times 2 = 8$$


$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

2 four times  
Or  
4 groups of 2



## Progression From Mental to Written Methods for Multiplication

Be able to know 2, 5 and 10 times table  
 Be able to calculate 3, 4, 5, 6, 7, 8 and 9 times tables

5.

To be able to partition a 2-digit number into groups of ten and units

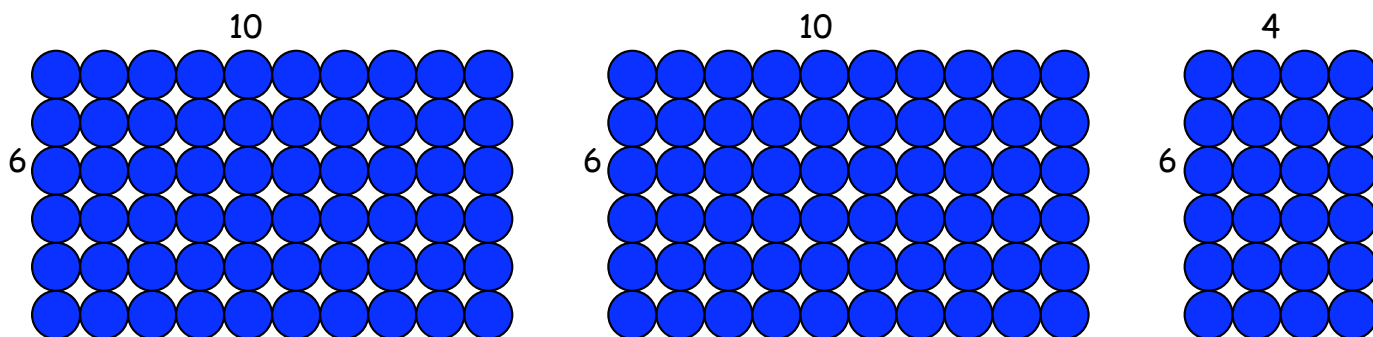
$$24 \times 6 = (10 + 10 + 4) \times 6$$

$$\begin{array}{r} 10 \times 6 = 60 \\ 10 \times 6 = 60 \\ 4 \times 6 = +24 \\ \hline 144 \end{array}$$

6.

To be able to use arrays to multiply TU X U

$$24 \times 6 =$$



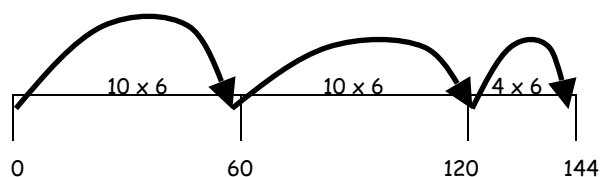
$$60 + 60 + 24 = 144$$

7.

Be able to use empty number lines to multiply TU by U

$$24 \times 6 = 144$$

$$(20 \times 6) + (4 \times 6)$$



## Progression From Mental to Written Methods for Multiplication

Be able to partition any 2-digit, and then 3-digit number

Know or be able to calculate quickly all tables to x10

Be able to multiply by a multiple of 10

8. Multiply TU x U using the grid method

$$24 \times 6 = 144$$

x		20		4
6		120		24

HTU  
120  
+ 24  
-----  
144

Be able to partition any 3-digit number into H T U

$$124 \times 6 = 744$$

x		100		20		4
6		600		120		24

HTU  
600  
120  
+ 24  
-----  
744

## Progression From Mental to Written Methods for Multiplication

Be able to round numbers to the nearest 10 or 100

9. Use approximation by rounding to check the answer is a sensible size

The answer will be approximately:  $21 \times 39 =$   
 $20 \times 40 = 800$

Be able to multiply any whole number by 10 or multiple of 10, 100 or multiple of 100

10. Multiply TU  $\times$  TU using the grid method

$$24 \times 36 = 864$$

x	20	4
30	600	120
6	120	24

$$\begin{array}{r}
 \text{HTU} \\
 600 \\
 120 \\
 120 \\
 + 24 \\
 \hline
 864
 \end{array}$$

Be able to multiply any decimal number by 10, 100 or 1000

11. Use grid method to multiply decimal number by U then TU

$$6.24 \times 36 = 224.64$$

x	6	0.2	0.04
30	180	6.0	1.2
6	36	1.2	0.24

$$\begin{array}{r}
 \text{HTU} . + \text{h} \\
 180.00 \\
 6.00 \\
 1.20 \\
 36.00 \\
 1.20 \\
 + 0.24 \\
 \hline
 224.64 \\
 \hline
 11
 \end{array}$$

## Progression From Mental to Written Methods for Division

*Prerequisite skill:*

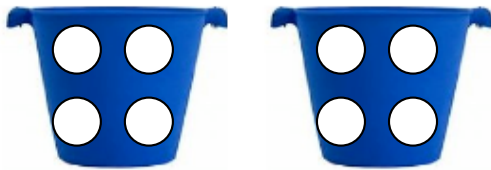
- Addition and subtraction need to be reached before attempting division.

1.

To be able to divide objects into equal groups of two or two equal groups



8 in four groups of 2s  
(groupings)

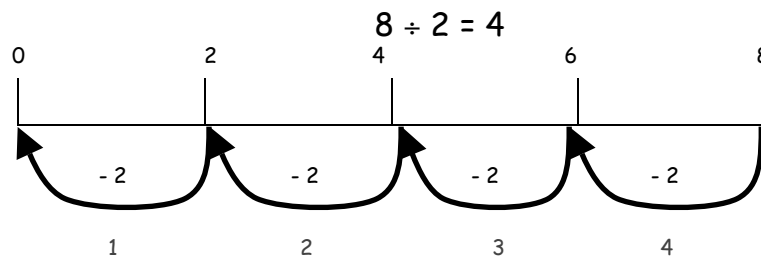


8 in two equal groups of 4s  
(sharing)

*Then equal groups of numbers*

2.

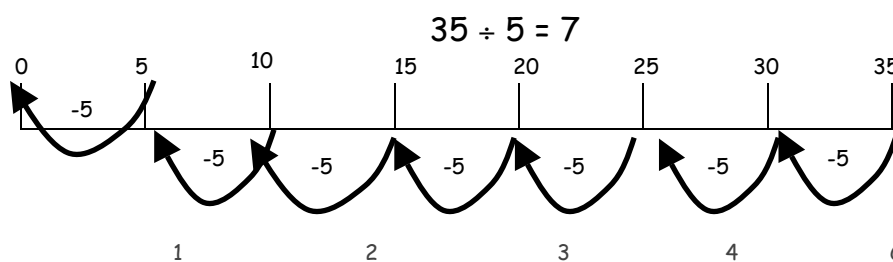
To use a number line to calculate how many jumps of 2, 5, or 10 are needed to reach a given number



Count the jumps of 2

3.

To understand division as repeated subtraction



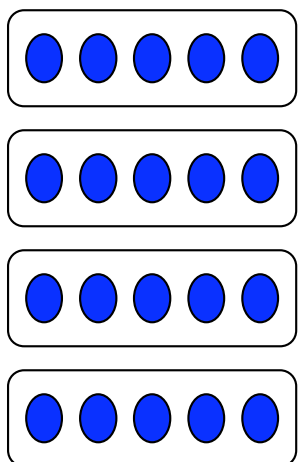
Count the jumps of 5

## Progression From Mental to Written Methods for Division

4.

To use arrays to solve division problems using mathematical terms, sharing and grouping

### Grouping



Four horizontal boxes, each containing five blue circles. The boxes are arranged vertically.

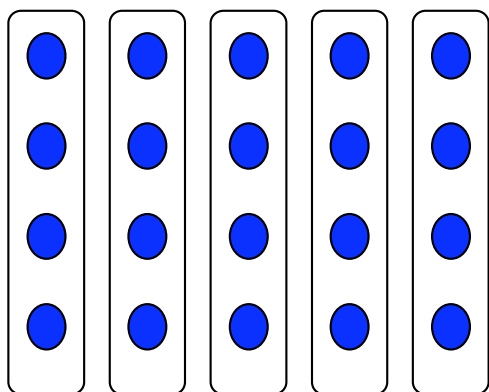
$5 \times 4 = 20$

4 groups of 5 = 20

$20 \div 5 = 4$

20 in groups of 5 gives 4 groups

### Sharing



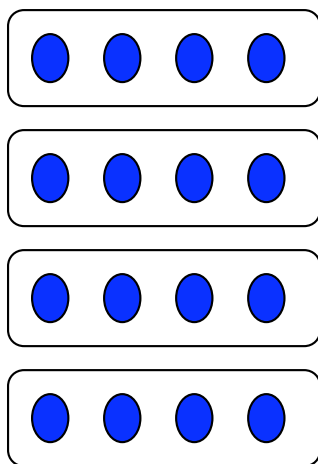
Five vertical boxes, each containing four blue circles. The boxes are arranged horizontally.

$4 \times 5 = 20$

5 groups of 4 = 20

$20 \div 5 = 4$

20 in 5 equal groups gives 4 in each group



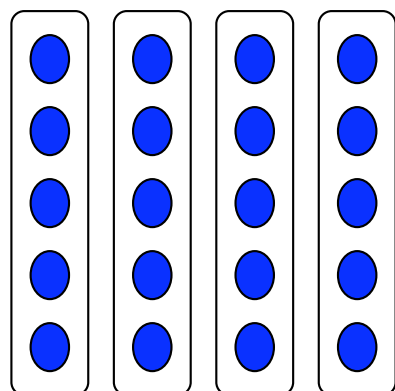
Four horizontal boxes, each containing four blue circles. The boxes are arranged vertically.

$4 \times 5 = 20$

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20 in groups of 4 gives 5 groups



Four vertical boxes, each containing five blue circles. The boxes are arranged horizontally.

$5 \times 4 = 20$

4 groups of 5 = 20

$20 \div 4 = 5$

20 in 4 equal groups gives 5 in each group

*These concepts are best taught practically*

Understand the inverse relationship of multiplication and division

## Progression From Mental to Written Methods for Division

5.

To understand how to deal remainders

They need to understand how to put the answer back into the context of the question.

### Grouping

A car holds 5 people.  
How many cars are needed to transport 17 people?

$$17 \div 5 = 3 \text{ r } 2$$

$$= 4 \text{ cars}$$

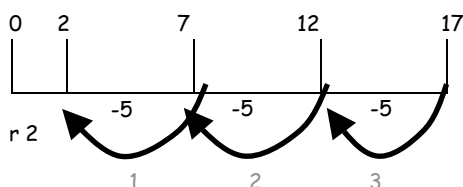
*(Group or round up)*

I can fit 5 chocolates into a box. I had 17 chocolates.  
How many full boxes will I have?

*(Group or round down)*

$$17 \div 5 = 3 \text{ r } 2$$

$$= 3 \text{ full boxes}$$



$$3 \text{ r } 2$$

### Sharing

Gran gave £17 to be shared between her 5 Grandchildren. How much does each child receive?

$$£17.00 \div 5 = £3.40$$

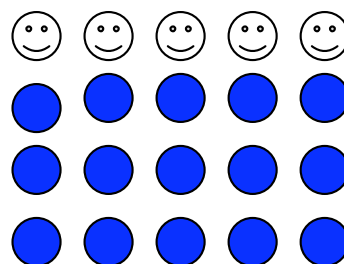
*(Whole number and decimal fraction)*

I share 17 lollies equally between 5 party bags. How many lollies are in each bag?

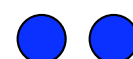
*(Whole number and remainders)*

$$17 \div 5 = 3 \text{ r } 2$$

$$= 3 \text{ lollies in each bag}$$



$$3 \text{ r } 2$$



To be efficient pupils need to:

- Know the 2, 5 and 10 times tables and be able to quickly calculate tables to x 10
- Be able to multiply by 100, 50 and 20 and then other multiples of 10.

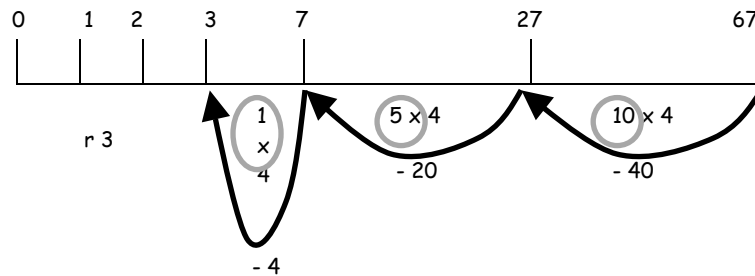
## Progression From Mental to Written Methods for Division

6.

Divide TU by U

Use larger multiples of the divisor on a number line - chunking

$$67 \div 4 = 16 \text{ r } 3$$



Circle the 'chunks' and add up mentally:  $10 + 5 + 1 = 16$  and remainder 3

Chunking using subtraction

$$\begin{array}{r}
 67 \div 4 = 16 \text{ r } 3 \\
 \underline{- 40} \quad (10 \times 4) \\
 27 \\
 \underline{- 20} \quad (5 \times 4) \\
 7 \\
 \underline{- 4} \quad (1 \times 4) \\
 \text{r } 3
 \end{array}$$

KEY FACTS:

$100 \times 4 = 400$   
 $10 \times 4 = 40$   
 $5 \times 4 = 20$   
 $2 \times 4 = 8$   
 $1 \times 4 = 4$

Circle the 'chunks' and add up mentally:  $10 + 5 + 1 = 16$  and remainder 3

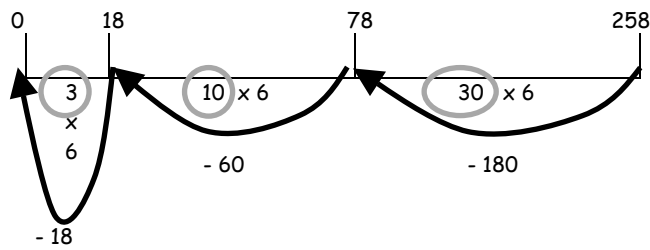
## Progression From Mental to Written Methods for Division

7.

Extend to larger numbers and decimals

Use larger multiples of the divisor on a number line - chunking

$$258 \div 6 = 43$$



Circle the 'chunks' and add up mentally:  $30 + 10 + 3 = 43$

Chunking using subtraction

$$\begin{array}{r}
 258 \div 6 = 43 \\
 - 120 \quad (20 \times 6) \\
 \hline
 138 \\
 - 120 \quad (20 \times 6) \\
 \hline
 18 \\
 - 18 \quad (3 \times 6) \\
 \hline
 0
 \end{array}$$

**KEY FACTS:**  
 $100 \times 6 = 600$   
 $50 \times 6 = 300$   
 $20 \times 6 = 120$   
 $10 \times 6 = 60$   
 $5 \times 6 = 30$   
 $2 \times 6 = 12$   
 $1 \times 6 = 6$

Circle the 'chunks' and add up mentally:  $20 + 20 + 3 = 43$

Pupils need to be able to use approximation in the process of 'keeping track' of where they are in the calculation

8.

Use decimal chunks to divide remainders

$$\begin{array}{r}
 34 \div 4 = 8.5 \\
 - 20 \quad (5 \times 4) \\
 \hline
 14 \\
 - 8 \quad (2 \times 4) \\
 \hline
 6 \\
 - 4 \quad (1 \times 4) \\
 \hline
 2 \\
 - 2 \quad (0.5 \times 4) \\
 \hline
 0
 \end{array}$$

**KEY FACTS:**  
 $10 \times 4 = 40$   
 $5 \times 4 = 20$   
 $2 \times 4 = 8$   
 $1 \times 4 = 4$   
 $0.5 \times 4 = 2$

Circle the 'chunks' and add up mentally:  $20 + 20 + 3 = 43$

## Methods for Solving Time Problems

*Prerequisite skill:*

- To understand that there are 60 minutes in an hour
- To be able to convert analogue time to digital time
- To know that after 60 they need to start the new set of minutes for the next hour

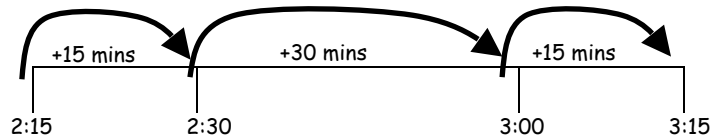
1.

To use a number line to find the difference between times

Question: How long is it between 2:15 and 3:15?

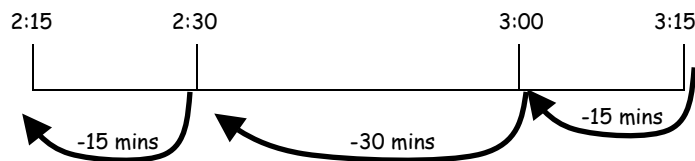
$$2:15 + \square = 3:15$$

*Counting on*



Add up the minute jumps:  $15 + 30 + 15 = 60$  minutes = 1 hour

*Counting back*



Add up the minute jumps:  $15 + 30 + 15 = 60$  minutes = 1 hour

Answer: There is 1 hour or 60 minutes between 2:15 and 3:15

Remember when solving worded problems, answer in the original context

## Progression From Mental to Written Methods for Time

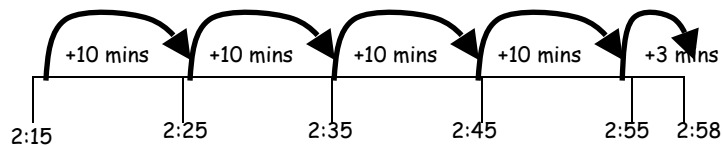
2.

To use a number line to solve addition problems

Question: The time now is 2:15pm. John spends 43 minutes jogging. What is the time when he finishes?

$$2:15 + 43 =$$

Addition



Answer: John finishes jogging at 2:58pm

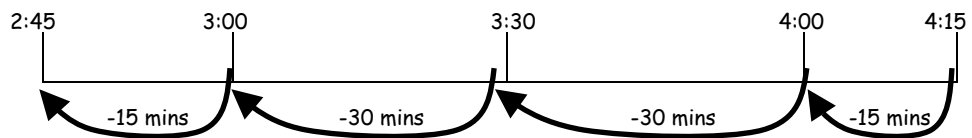
3.

To use a number line to solve subtraction problems

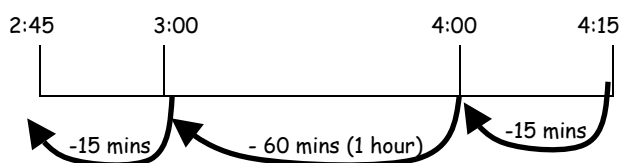
Question: The football match finished at 4:15pm. The match was 90 minutes long. What time did the match start?

Jumps can be made back in minutes

$$4:15 - 90 =$$



One jump can be made to the previous hour and then the remaining jumps in minutes.



Answer: The match started at 2:45 pm.

## Methods for solving Word Problems

To be able use the steps for success when solving worded problems

### Steps:

1. Read the question carefully
2. Underline or highlight the key numbers and words
3. Choose which operation to create a number sentence
4. Solve the problem using an appropriate method
5. Re-write the answer in the original context
6. Check your calculation/answer using the inverse operation or a different method

## Useful ITPs:

### Counting on and back (KS1, early KS2)

This ITP simulates a 100-bead string; it is suitable for addition and subtraction particularly with counting and early calculation strategies.

### Difference (KS1, KS2)

This ITP compares two of rows of beads and analyses the calculations they represent; it promotes the language of addition and subtraction, particularly the interpretation of difference as 'jumps'.

### Number Facts (KS1)

This ITP models addition by combining and counting; it also models subtraction by partitioning and taking away.

### Number Grid (KS1, KS2)

This ITP generates a 100 square; it also explores different multiples, number sequences and patterns on grids of different sizes.

### Number line (KS1, early KS2)

This ITP generation random numbers quickly; it looks at the disturbing of sums and differences between numbers generated; useful for adding several small numbers.

### Ordering Numbers (KS1, early KS2)

This ITP supports counting and recording skills; it also supports addition and subtraction activities moving from counters to the number line.

### Place Value (KS1, KS2)

This ITP has on screen place value cards; it demonstrates the construction and partitioning of numbers to secure children's understanding of place value. Partitioning numbers and rearranging them can be used to help children understand how pencil and paper methods of calculation are recorded.